

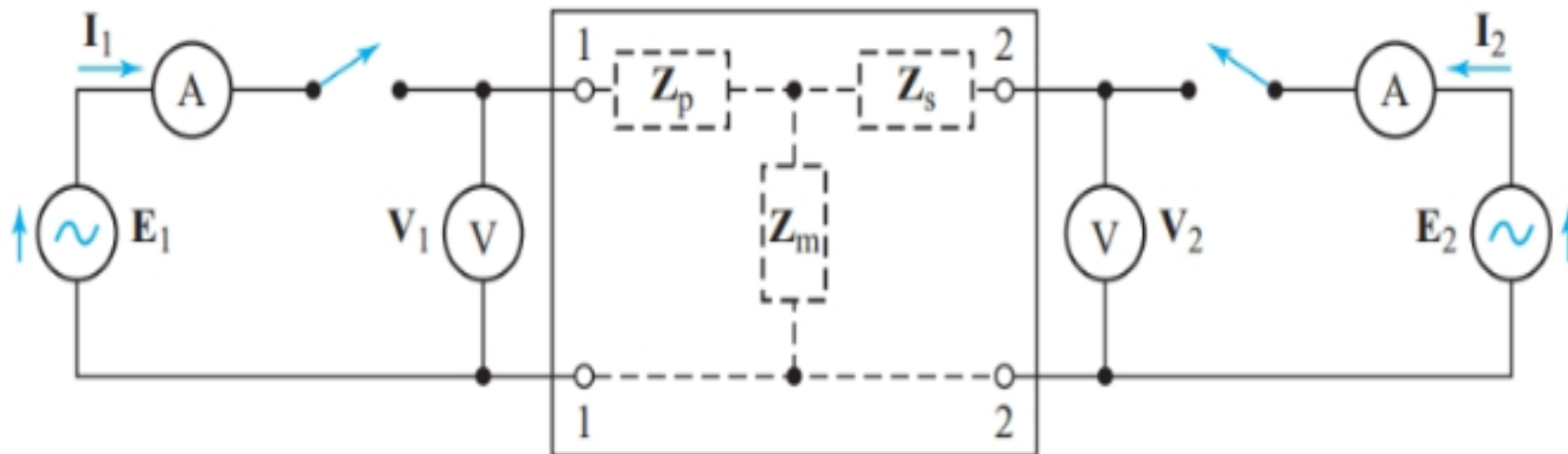
# SCATTERING PARAMETERS (S-PARAMETERS)

**EEEN 566 – MICROWAVE ENGINEERING**

**Friday, 13 February 2026**

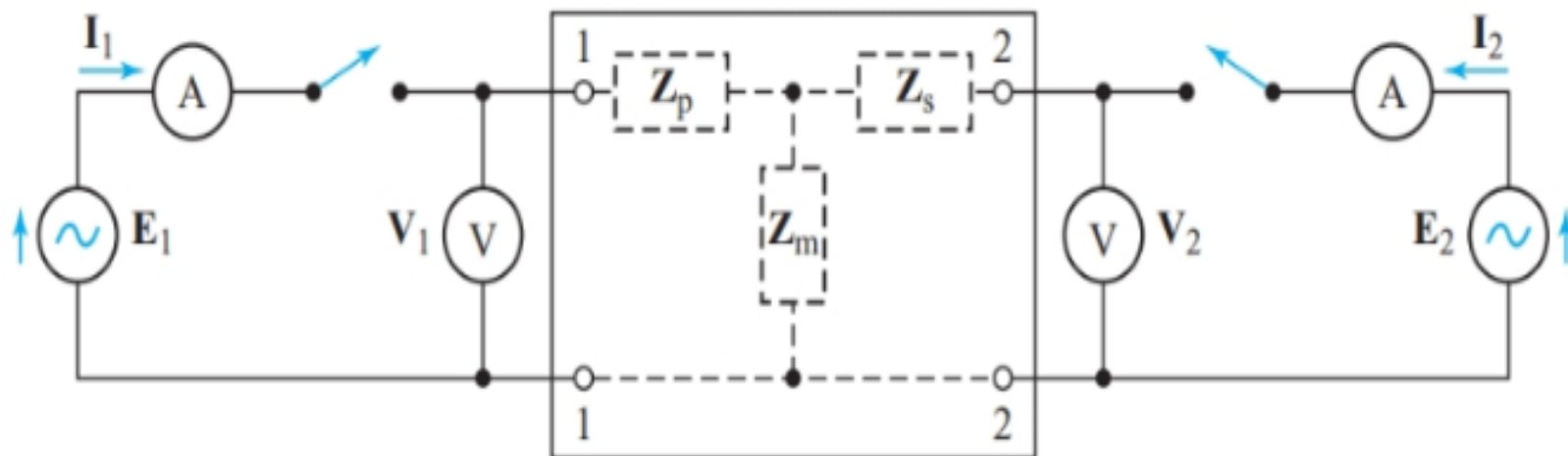
# PROBLEMS WITH USING Z-PARAMETERS AT MICROWAVE FREQUENCIES

- Z-parameters (open-circuit impedance parameters)** are determined by measuring the input and output voltages and currents under open-circuit conditions.
- Problems:**
  - It is not possible to measure currents and voltages in microwave components.
  - Open-circuit conditions cause instability and oscillations in High-frequency Circuits.
  - Open-circuit conditions cannot be realized in some microwave components, e.g. waveguides.



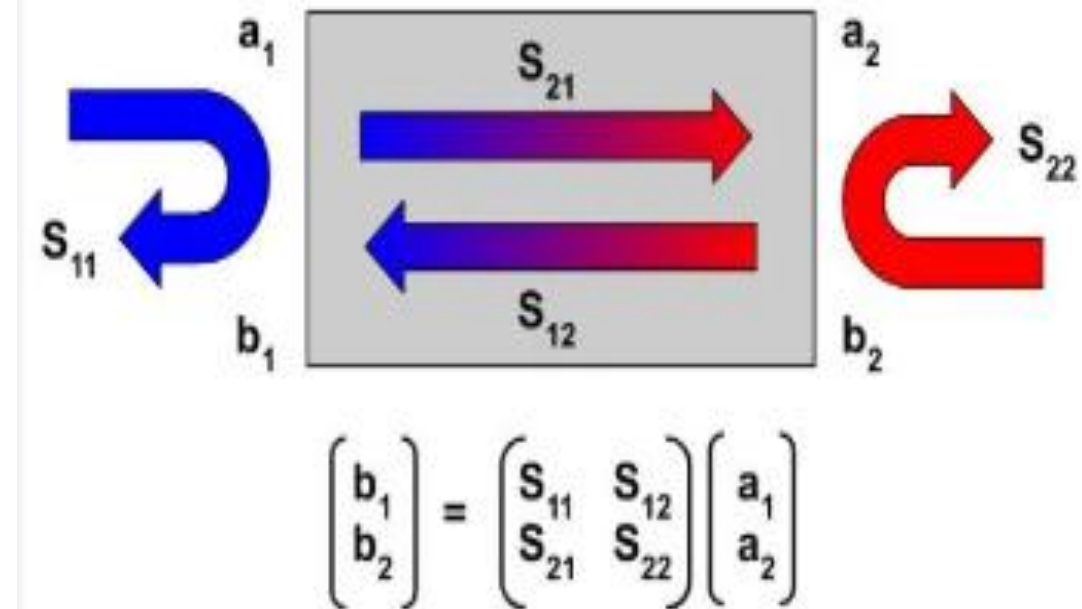
# PROBLEMS WITH USING Y-PARAMETERS AT MICROWAVE FREQUENCIES

1. **Y-parameters (Short-circuit admittance parameters)** are determined by measuring the input and output voltages and currents under short-circuit conditions.
2. **Problems:**
  - a) It is not possible to measure currents and voltages in microwave components.
  - b) Short-circuit conditions cause instability and oscillations in High-frequency Circuits.
  - c) Short-circuit conditions cannot be realized in some microwave components, e.g. waveguides



# WHAT ARE SCATTERING PARAMETERS?

1. **Scattering matrix (s-parameters)** is a mathematical matrix that describes the power relationships between different ports of a microwave network.
2. **S-parameters** detail how much of an incident signal is reflected back at a port or transmitted to other ports.
3. **S-parameters** are crucial for analyzing microwave circuits and components where direct voltage and current measurements are difficult at high frequencies.



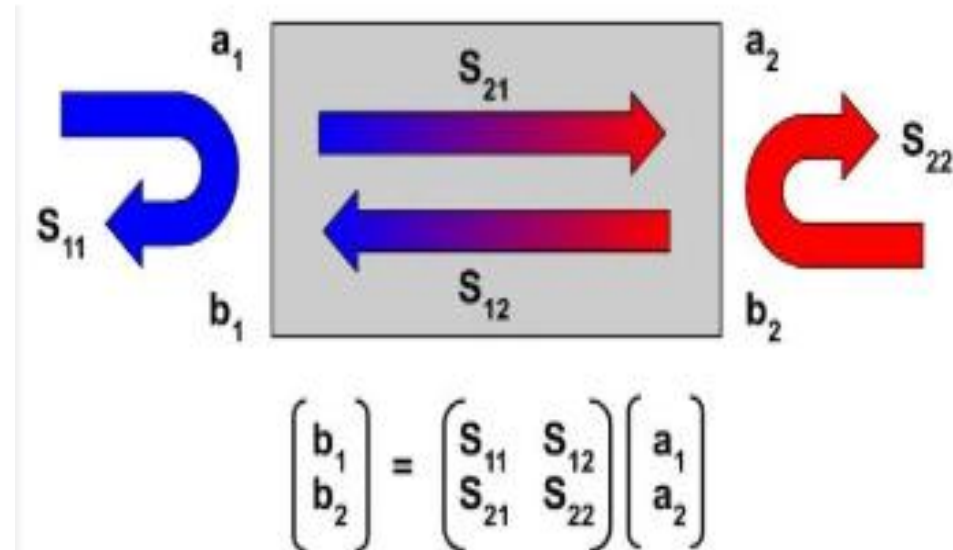
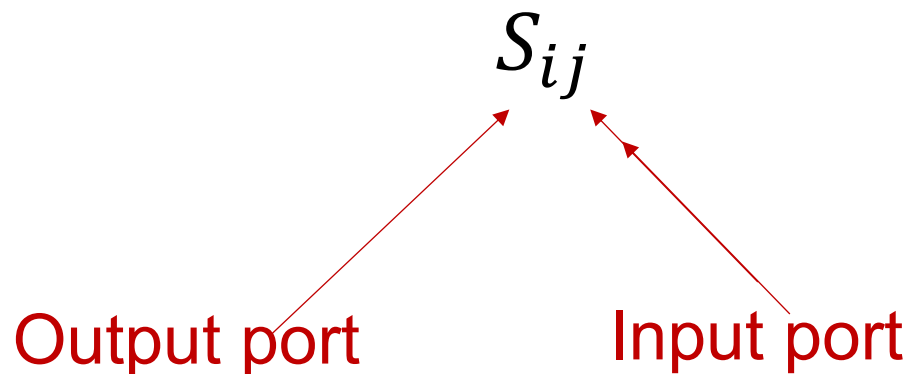
# S-PARAMETER TERMINOLOGY

## Incident & Reflected Values

$a$  - RMS value of incident wave  
 $b$  - RMS value of reflected wave



## S-Matrix Values



# GENERAL FORM OF S-PARAMETERS

The general form of S-parameters for 2-port network is

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

We can write the relationship as

$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}$$

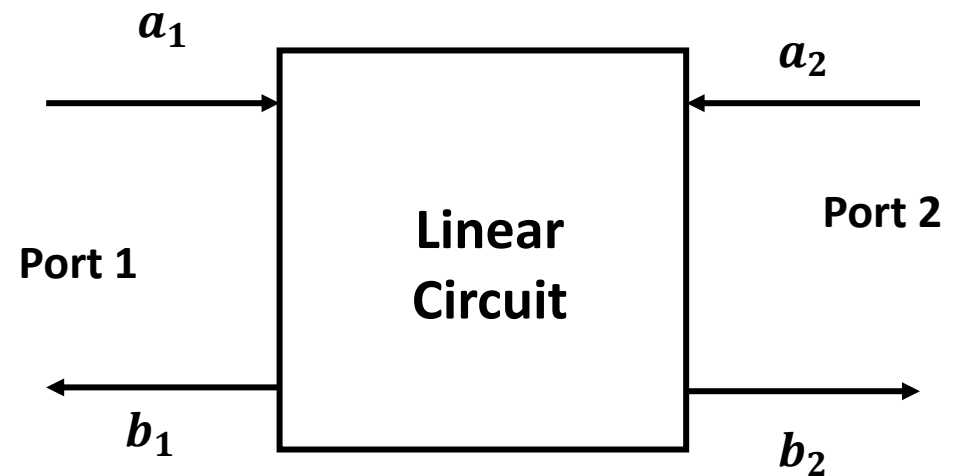
where

$a_1$  = power of incident wave port 1

$a_2$  = power of incident wave port 2

$b_1$  = power of reflected wave port 1

$b_2$  = power of reflected wave port 2



# INTERPRETATION OF S-PARAMETERS

$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}$$

$$S_{11} = \frac{b_1}{a_1} \quad \left| \quad a_2 = 0 \right.$$

(a) Input Reflection coefficient

$$S_{12} = \frac{b_1}{a_2} \quad \left| \quad a_1 = 0 \right.$$

(b) Forward transmission coefficient

$$S_{21} = \frac{b_2}{a_1} \quad \left| \quad a_2 = 0 \right.$$

(c) Reverse transmission coefficient

$$S_{22} = \frac{b_2}{a_2} \quad \left| \quad a_1 = 0 \right.$$

(d) Output Reflection coefficient

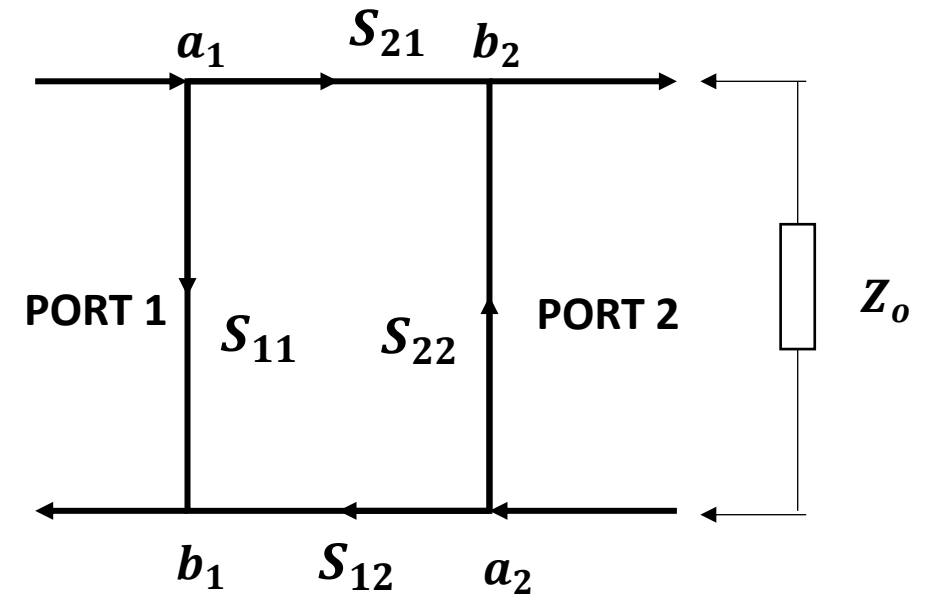
# TERMINATING PORT 2 ITS CHARACTERISTIC IMPEDANCE

If we terminate port 2 with its characteristic impedance, i.e  $S_{22} = 0$  and  $S_{12} = 0$

We get:

$$b_1 = a_1 S_{11}$$

$$b_2 = a_1 S_{21}$$



$$b_1 = a_1 S_{11} + \cancel{a_2 S_{12}}$$

$$b_2 = a_1 S_{21} + \cancel{a_2 S_{22}}$$

# TERMINATING PORT 2 ITS CHARACTERISTIC IMPEDANCE

If we terminate port 2 with its characteristic impedance ( $S_{22} = 0, S_{12} = 0$ ) and apply a signal at port 1, we get:

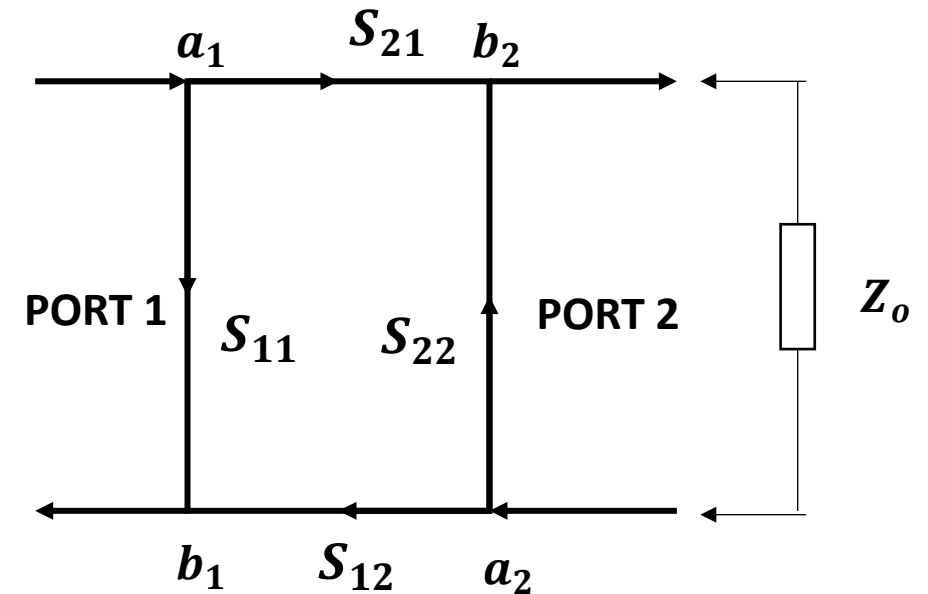
$$b_1 = a_1 S_{11}$$

$$b_2 = a_1 S_{21}$$

Then

$$S_{11} = \frac{b_1}{a_1}$$

$$S_{21} = \frac{b_2}{a_1}$$



$$b_1 = a_1 S_{11} + a_2 S_{12}$$

$$b_2 = a_1 S_{21} + a_2 S_{22}$$

# TERMINATING PORT 1 ITS CHARACTERISTIC IMPEDANCE

Similarly, if we terminate port 1 with its characteristic impedance ( $S_{11} = 0, S_{21} = 0$ ) and apply a signal to port 2.

We get

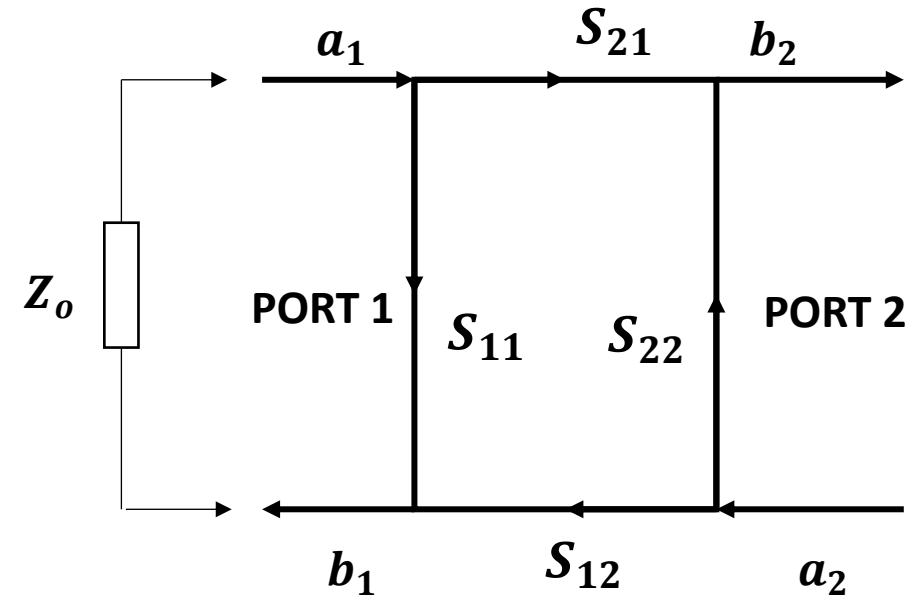
$$b_1 = a_2 S_{12}$$

$$b_2 = a_2 S_{22}$$

Then

$$S_{12} = \frac{b_1}{a_2}$$

$$S_{22} = \frac{b_2}{a_2}$$



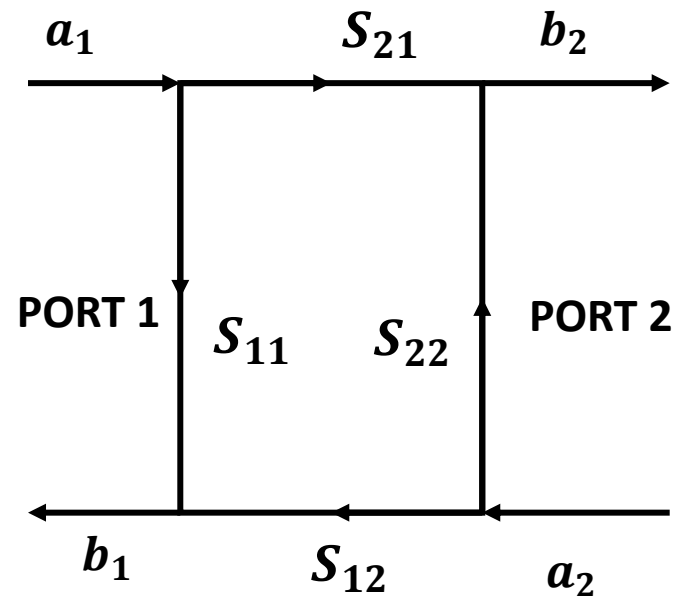
$$b_1 = a_1 S_{11} + a_2 S_{12}$$

$$b_2 = a_1 S_{21} + a_2 S_{22}$$

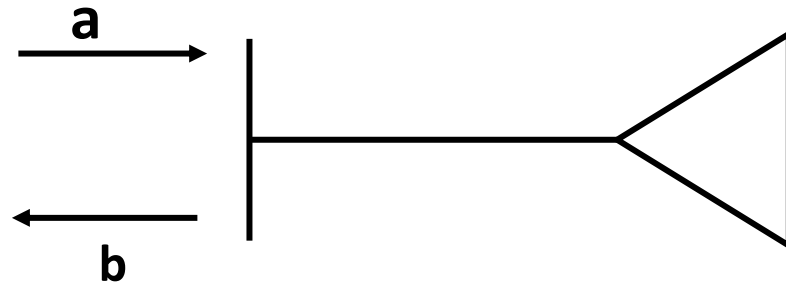
# S-MATRIX FOR 2-PORT DEVICE

The scattering matrix relates the output (reflected) waves  $[b_1, b_2]$  to the input (reflected) waves  $[a_1, a_2]$

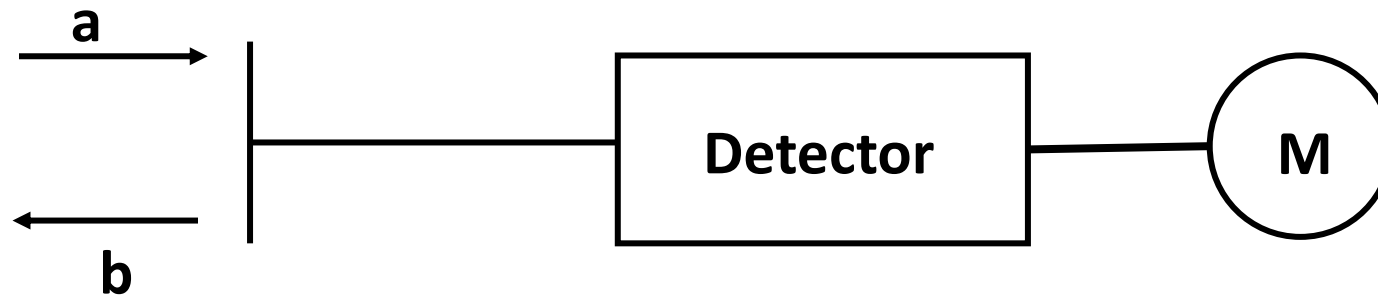
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



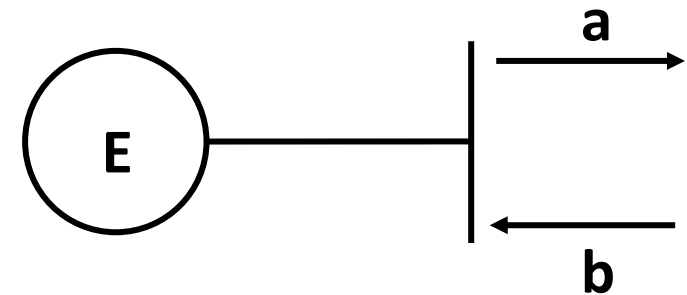
# FLOW GRAPH REPRESENTATION OF ONE-PORT DEVICES



(a) Termination



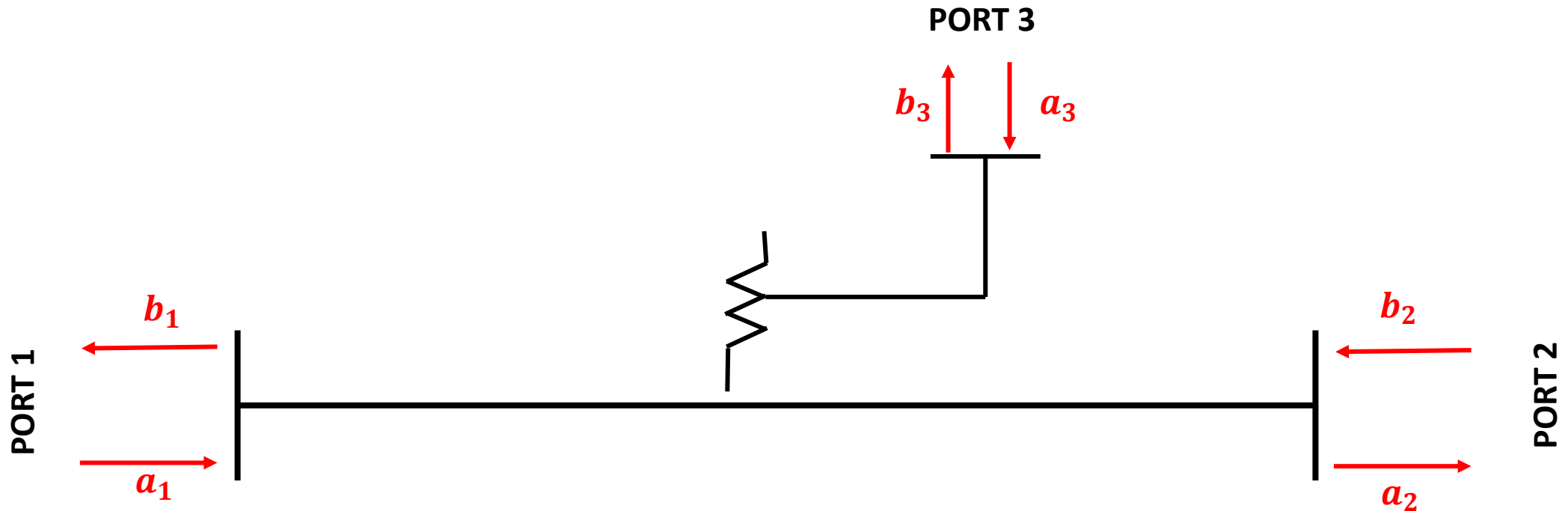
(b) Detector



(c) Signal Source

# THREE PORT DIRECTIONAL COUPLER

A three port directional coupler operates as a power divider in the forward direction.

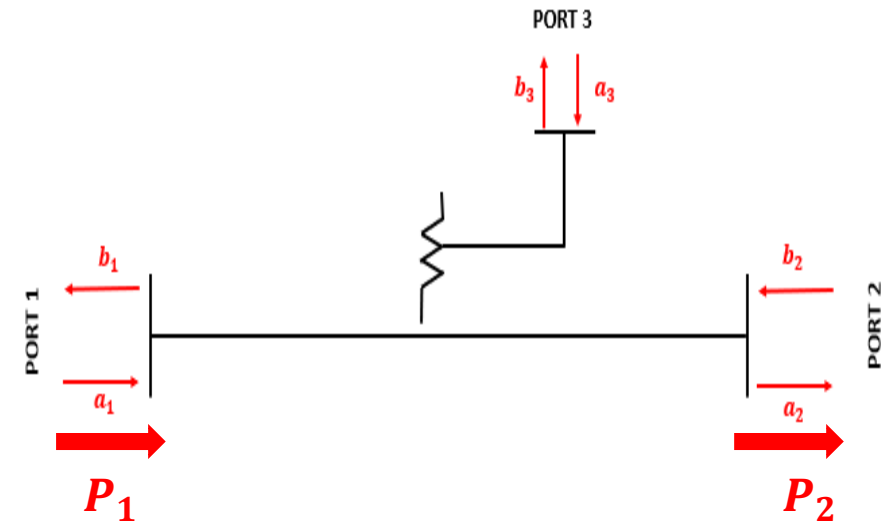


# COUPLING FACTOR FOR A 3-PORT COUPLER

Assuming that the wave is propagating from port 1 to port 2 in the primary line, the coupling factor and the directivity are defined, as

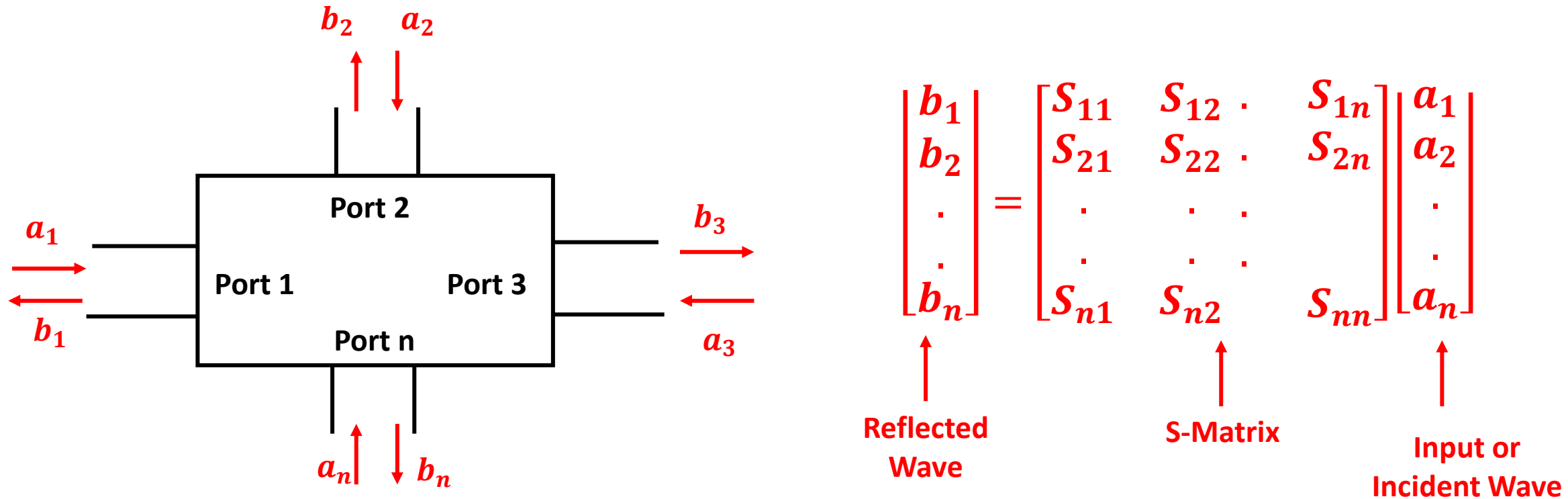
$$\text{Coupling factor} = 10 \log_{10} \left( \frac{P_2}{P_1} \right)$$

For example, a 3dB directional coupler divides power from port 1 into two equal parts to port 2 and port 3.



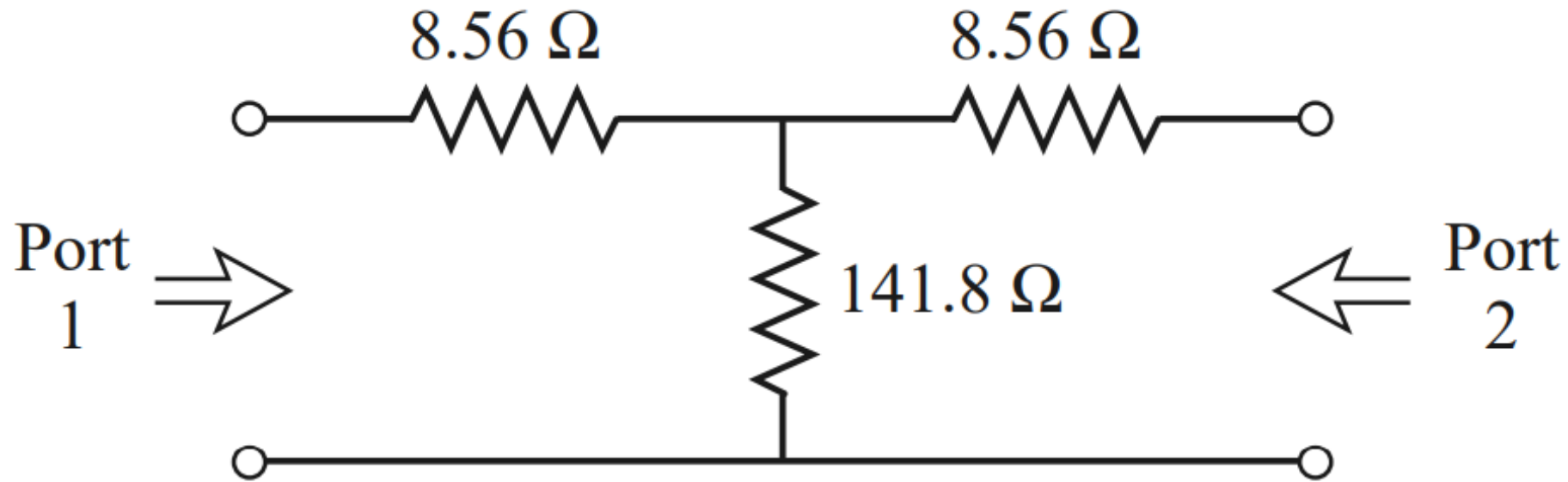
# S-PARAMETERS FOR AN N-PORT NETWORK

- During the analysis of power divider/coupler microwave systems, signals are represented as travelling waves with associated power levels.
- Power relations between different ports in the microwave system are defined by using scattering parameters or s-parameters as shown.



# WORKED EXAMPLE

Find the scattering parameters of the 3 dB attenuator circuit with 50 Ohms characteristic impedance shown below assuming



# WORKED EXAMPLE SOLUTION /01

When port 2 is terminated with its characteristic impedance, we have

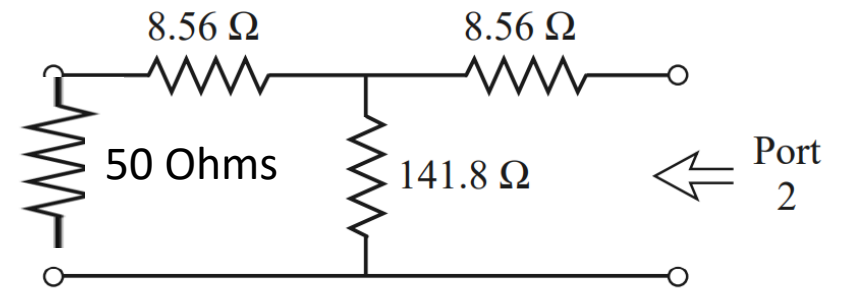
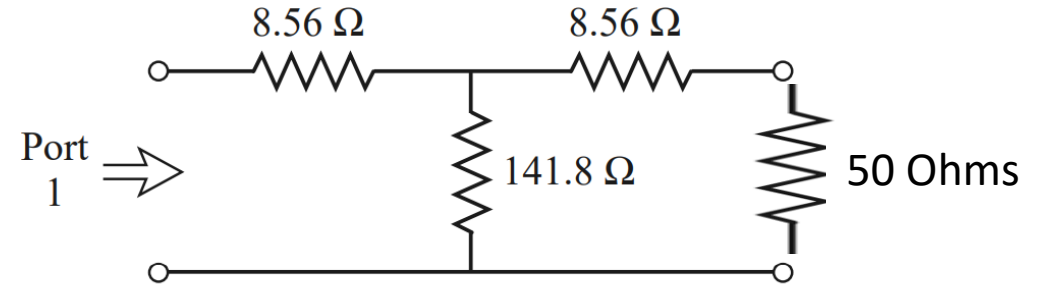
$$S_{11} = \frac{V_1^-}{V_1^+} \text{ when } V_2^+ = 0$$

$$S_{11} = \frac{Z_{in}^1 - Z_o}{Z_{in}^1 + Z_o}$$

$$\text{But } Z_{in}^1 = 8.56 + \frac{141.8(8.56 + 50)}{141.8 + 8.56 + 50} = 50$$

$$\text{Hence } S_{11} = 0$$

The circuit is symmetric, therefore  $S_{11} = S_{22} = 0$



# WORKED EXAMPLE SOLUTION /02

$$v_2 = v_1 \left( \frac{41.44}{41.44 + 8.56} \right) \left( \frac{50}{50 + 8.56} \right) \\ = 0.707v_1$$

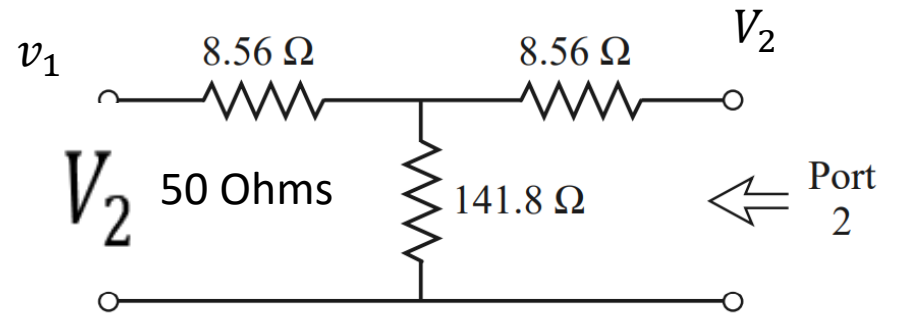
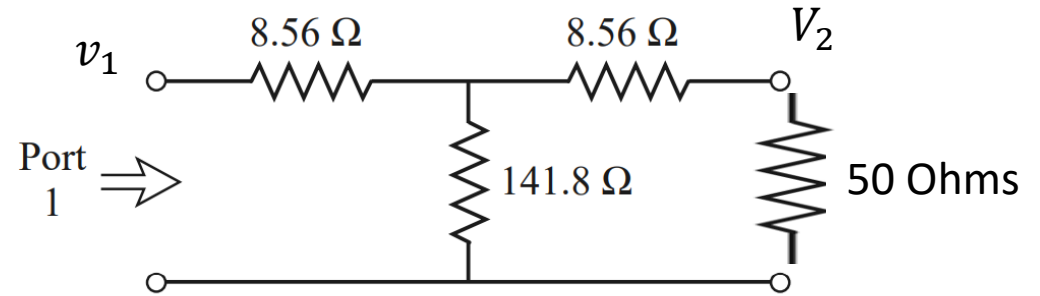
Similarly, if port 1 is terminated with its  $Z_o$ , we get:

$$v_1 = v_2 \left( \frac{41.44}{41.44 + 8.56} \right) \left( \frac{50}{50 + 8.56} \right) \\ = 0.707v_2$$

Hence  $S_{12} = S_{21} = 0.707$

The S matrix is therefore:

$$S = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$



# APPLICATION OF S-PARAMETERS

1. Applications of S-parameters are:
  - a) Characterizing the performance of microwave and RF circuits.
  - b) Designing filters, amplifiers, and other microwave components.
  - c) Predicting system performance by cascading S-parameters of different devices.